

Math 3008 Proof of

$$E(X+Y) = E(X) + E(Y)$$

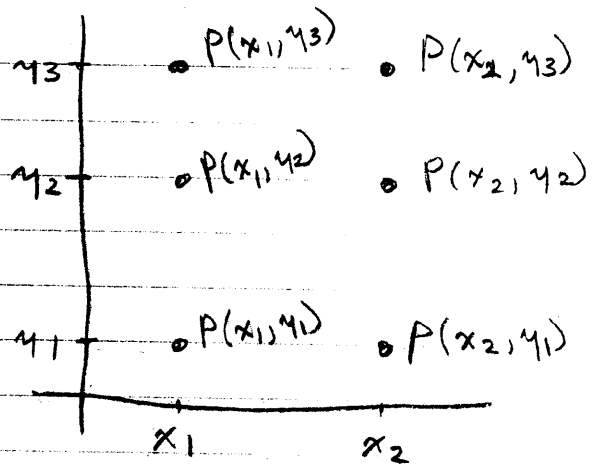
Let  $P(X=x_i \text{ and } Y=y_j) = P(x_i, y_j)$

$$P(X=x_i) = P(x_i, y_1) + P(x_i, y_2) + P(x_i, y_3)$$

$$= \sum_{j=1}^3 P(x_i, y_j)$$

$$P(Y=y_j) = P(x_1, y_j) + P(x_2, y_j)$$

$$= \sum_{i=1}^2 P(x_i, y_j)$$



$$E(X+Y)$$

$$= \sum_{i,j} (x_i + y_j) \cdot P(x_i, y_j)$$

$$= \sum_{i=1}^2 \sum_{j=1}^3 (x_i + y_j) \cdot P(x_i, y_j)$$

$$= \sum_{i=1}^2 \sum_{j=1}^3 [x_i \cdot P(x_i, y_j) + y_j \cdot P(x_i, y_j)]$$

$$= \sum_{i=1}^2 \sum_{j=1}^3 x_i P(x_i, y_j) + \sum_{i=1}^2 \sum_{j=1}^3 y_j P(x_i, y_j)$$

$$= \sum_{i=1}^2 x_i \sum_{j=1}^3 P(x_i, y_j) + \sum_{j=1}^3 y_j \sum_{i=1}^2 P(x_i, y_j)$$

$$= \sum_{i=1}^2 x_i P(X=x_i) + \sum_{j=1}^3 y_j P(Y=y_j)$$

$$= E(X) + E(Y)$$

$$= (x_1 + y_1) P(x_1, y_1) + (x_2 + y_1) P(x_2, y_1)$$

$$+ (x_1 + y_2) P(x_1, y_2) + (x_2 + y_2) P(x_2, y_2)$$

$$+ (x_1 + y_3) P(x_1, y_3) + (x_2 + y_3) P(x_2, y_3)$$

$$= x_1 P(x_1, y_1) + x_2 P(x_2, y_1)$$

$$+ x_1 P(x_1, y_2) + x_2 P(x_2, y_2)$$

$$+ x_1 P(x_1, y_3) + x_2 P(x_2, y_3)$$

$$+ y_1 P(x_1, y_1) + y_1 P(x_2, y_1)$$

$$+ y_2 P(x_1, y_2) + y_2 P(x_2, y_2)$$

$$+ y_3 P(x_1, y_3) + y_3 P(x_2, y_3)$$

$$= x_1 [P(x_1, y_1) + P(x_1, y_2) + P(x_1, y_3)]$$

$$+ x_2 [P(x_2, y_1) + P(x_2, y_2) + P(x_2, y_3)]$$

$$+ y_1 [P(x_1, y_1) + P(x_2, y_1)]$$

$$+ y_2 [P(x_1, y_2) + P(x_2, y_2)]$$

$$+ y_3 [P(x_1, y_3) + P(x_2, y_3)]$$

$$= x_1 P(X=x_1) + x_2 P(X=x_2)$$

$$+ y_1 P(Y=y_1) + y_2 P(Y=y_2) + y_3 P(Y=y_3)$$

$$= E(X) + E(Y)$$