

DENSITY PROBLEMS

From Peggy Tang Strait, Prob. + Stat.
with Applications, 2nd ed, p. 150

1. 1.1467, .8533, .3733, 0, 0, 0; 2. (b) .32, .68, .16, .42; 3. (a) $5/32 = .15625$; (b) $11/32 = .34375$; (c) $.21875$; (d) $.125$; (e) 0; 4. (a) $.7407$; (b) $.2593$; (c) $.4259$; (d) $.5$; 5. a, c, d, e, f, and g are true statements.

Section 5.4

EXAMPLE 5.5 A THIRD TYPE OF RANDOM VARIABLE ("Mixed")

The following is an example of a random variable that is neither discrete nor continuous.

Consider the following experiment. Toss a coin. If the outcome is heads, the player receives 10 ounces of beer. If the outcome is tails, the player presses a button and receives anywhere from 0 to 10 ounces of beer. Let X be the amount of beer received by the player in this experiment. Explain why X is neither a discrete nor a continuous random variable.

SOLUTION

The set of possible values of X is the entire interval of real numbers from 0 to 10. Therefore, X cannot be a discrete random variable.

The probability of heads on the toss of the coin is $1/2$. Therefore, $P(X = 10) \geq 1/2$. Consequently X cannot be a continuous random variable because if X were continuous, then $P(X = 10) = 0$.

Exercises on density functions.

Hand in: Sketch the graph of each density. Shade the area(s),

1. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} (2/3)x & \text{for } 1 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find $P(X \leq 1.2)$, $P(X > 1.2)$, $P(1.2 \leq X \leq 1.6)$, $P(X = 1.5)$, $P(X \leq .2)$, and $P(X > 3)$.

2. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} -4x & \text{for } -.5 < x < 0 \\ 4x & \text{for } 0 < x < .5 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Sketch the probability density curve.
(b) Find $P(X \leq -.3)$, $P(X \leq .3)$, $P(-.2 \leq X \leq .2)$, $P(X > .2)$.

3. Sixteen-ounce boxes of cereal packed automatically by a certain machine are sometimes overweight and sometimes underweight. The actual weight in ounces over or under 16 is a random variable X whose probability density function is

$$f(x) = \begin{cases} (3/4)(1 - x^2) & \text{for } -1 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

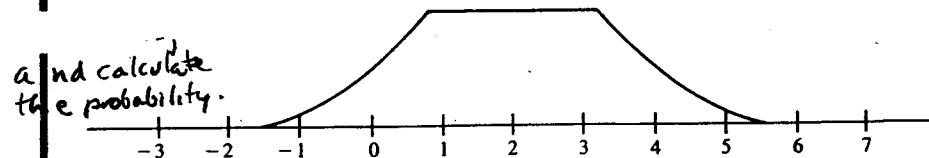
Negative values pertain to weight in ounces under the intended 16. Find the probability that a box of cereal packed by this machine will be

- (a) at least .5 ounces overweight;
(b) neither underweight nor more than .5 ounces overweight;
(c) neither overweight nor less than .3 ounces underweight; *< nuclear?*
(d) between .3 and .5 ounces underweight;
(e) exactly 16 ounces.
4. The pressure (measured in pounds per cm^2) at a certain valve is a random variable X whose probability density function is

$$f(x) = \begin{cases} (6/27)(3x - x^2) & \text{for } 0 < x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability that the pressure at this valve is

- (a) not more than 2 pounds per cm^2 ;
(b) greater than 2 pounds per cm^2 ;
(c) between 1.5 and 2.5 pounds per cm^2 ;
(d) less than 1.5 pounds per cm^2 .
5. Suppose that the figure that follows is a graph of the probability density function of the continuous random variable X .



Which of the following are true statements?

- (a) $P(X > 2) = P(X < 2)$.
(b) $P(X = 2) > P(X = 0)$.
(c) It is more likely that X will take on a value near 2 than near 4.
(d) $P(X > 6) = 0$.
(e) $P(X < 7) = 1$.
(f) It is unlikely that X will take on values greater than 5.
(g) The possible values of X between 1 and 3 are "uniformly distributed"