Math 251

Day 4 HW

B: Prove: If $x^* = a + bx$, then $\bar{x}^* = a + b\bar{x}$, $s^* = bs$

1) Prove $\boxed{\bar{x}^* = a + b\bar{x}}$

Proof:

for $n = 3$ | In general

$$\bar{x}^* = \frac{x_1^* + x_2^* + x_3^*}{3} \quad = \quad \frac{1}{n}\sum_{i=1}^{n} x_i^*$$

substitute $(a+bx_i)$ for x_i^*
$$= \frac{(a+bx_1)+(a+bx_2)+(a+bx_3)}{3} \quad = \quad \frac{1}{n}\sum_{i=1}^{n}(a+bx_i)$$

Using associative + distributive laws
$$= \frac{3a + b(x_1+x_2+x_3)}{3} \quad = \quad \frac{1}{n}\left[\sum_{i=1}^{n} a + \sum_{i=1}^{n} bx_i\right] = \frac{1}{n}\left[n \cdot a + b\sum_{i=1}^{n} x_i\right]$$

fraction rules
$$= a + b\frac{(x_1+x_2+x_3)}{3} \quad = \quad a + b\frac{\sum_{i=1}^{n} x_i}{n}$$

$\bar{x} = \frac{\sum x_i}{n}$
$$= a + b\bar{x} \quad = \quad a + b\bar{x}$$

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2) Prove $\boxed{s^* = bs}$. Note $s^* = \sqrt{\frac{\sum(x_i^* - \bar{x}^*)^2}{n-1}}$, $s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$

Proof:

for $n = 3$ | In general

$$s^* = \sqrt{\frac{(x_1^*-\bar{x}^*)^2 + (x_2^*-\bar{x}^*)^2 + (x_3^*-\bar{x}^*)^2}{3-1}} \quad = \quad \sqrt{\frac{\sum_{i=1}^{n}(x_i^* - \bar{x}^*)^2}{n-1}}$$

Substitute $x_i^* = a + bx_i$ and $\bar{x}^* = a + b\bar{x}$
$$= \sqrt{\frac{[(a+bx_1)-(a+b\bar{x})]^2 + [(a+bx_2)-(a+b\bar{x})]^2 + [(a+bx_3)-(a+b\bar{x})]^2}{3-1}} \quad = \quad \sqrt{\frac{\sum[(a+bx_i)-(a+b\bar{x})]^2}{n-1}}$$

a's cancel. Factor b's
$$= \sqrt{\frac{[b(x_1-\bar{x})]^2 + [b(x_2-\bar{x})]^2 + [b(x_3-\bar{x})]^2}{3-1}} \quad = \quad \sqrt{\frac{\sum[b(x_i-\bar{x})]^2}{n-1}}$$

Pull b thru the square
$$= \sqrt{\frac{b^2(x_1-\bar{x})^2 + b^2(x_2-\bar{x})^2 + b^2(x_3-\bar{x})^2}{3-1}} \quad = \quad \sqrt{\frac{\sum b^2(x_i-\bar{x})^2}{n-1}}$$

Factor b
$$= \sqrt{b^2\,\frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^2}{3-1}} \quad = \quad \sqrt{\frac{b^2\sum(x_i-\bar{x})^2}{n-1}}$$

Pull b thru $\sqrt{\ }$, substitute s.
$$= b\sqrt{\frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^2}{3-1}} \quad = \quad b\sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}}$$

$$= bs. \quad = \quad bs$$

Day 4

Homework solutions -

1.141ab: Find the linear transformation $x_{new} = a + bx$

	mean	s.d.	mean	s.d.
new	100	20	100	20
old (raw)	75	10	82	11
	3rd grade		6th grade	

Since $\bar{x}_{new} = a + b\bar{x}$, $s_{new} = bs$ } we substitute for $\bar{x}_{new}$, $\bar{x}$, s_{new}, s, and get 2 equations in 2 unknowns, $a + b$. Solve for a and b

3rd grade:

$$\begin{cases} 100 = a + b \cdot 75 \\ 20 = b \cdot 10 \end{cases}$$

$b = 2$

$100 = a + 2 \cdot 75$

$= a + 150$

$a = -50$

$x_{new} = -50 + 2x$

6th grade

$$\begin{cases} 100 = a + b \cdot 82 \\ 20 = b \cdot 11 \end{cases}$$

$b = 20/11 =$

$100 = a + 20/11 \cdot 82$

$100 = a + 149.09$

$a = -49.09$

$x_{new} = -49.09 + 1.818x$