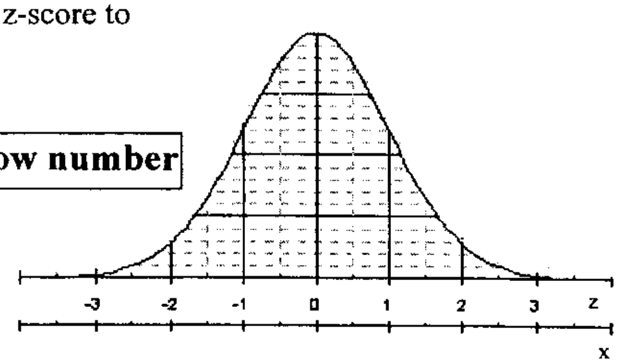
Normal Probability Practice.

When data follows the normal distribution, three kinds of numbers are in play:

- ***"Raw"** number--**x**--the number the data is originally measured in (* $, inches, days, sometimes percents ☹...) This is always a number on the horizontal (x) axis.
- **z-score**-z--the place a particular number sits on the x-axis, but measured in the z-scale: standard deviations from the mean (+ is above, – is below)
- **Proportion** = **area**. (below /above the number) The proportion below the number, written in percent, is the "Percentile" for that number. If 59% of cats weigh less than 18 pounds, then 18 pounds is the 59th percentile of cat weights.

Most questions require moving from one kind of number to another.
You always need the **mean** μ and **standard deviation** σ (in raw units)
With **Standard Normal Table** you must move through the z-score to get between raw and proportion. The most common printed (and computerized) tables, relate z-score and proportion *below* the z-score.



***Raw number ←→ z-score ←→ Proportion below number**

z-score from raw:

$$z = \frac{x-\mu}{\sigma} \qquad z = \frac{\text{raw} - \text{mean}}{\text{s.d.}}$$

Raw from z-score:

$x = \mu + z\sigma$ raw = z s.d.'s above the mean = mean + z · (s.d.)

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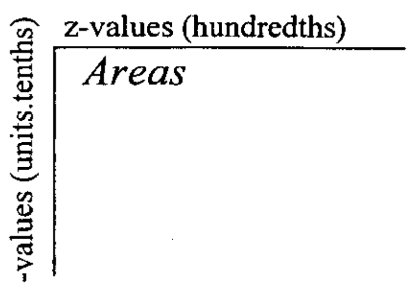
*z-score to Proportion **below** it:*
Find z on edges of table, read area inside table.

*Proportion **below** a number to its z-score:*
Find closest area inside table, read z from edge(s).



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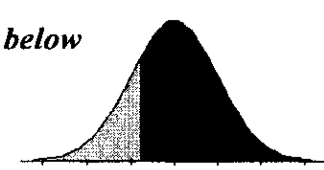
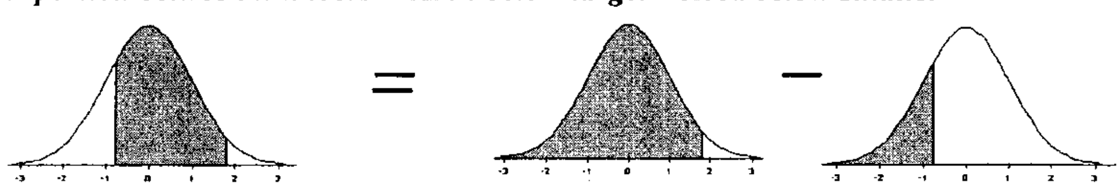
*Proportion **above** ←→ Proportion **below***
1 – Area below = Area above
1 – Area above = Area below



Proportion between numbers = Area below larger – Area below smaller



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Keeping track:

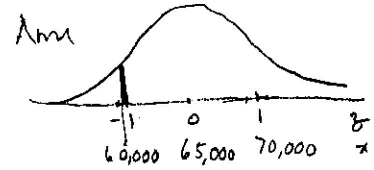
Label raw values with * *give units.
z-scores (oval)
Area/proportion (box)

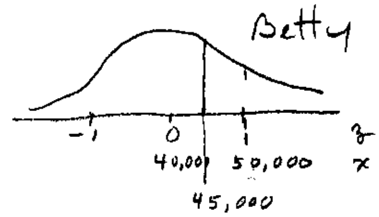
READ IT!
Mark every number *Raw (z) [Area/Prop.] *=$ here

4) Ann lives in the Plumtown suburb of Metropolis, where the distribution of incomes is approximately normal with mean $65,000 and standard deviation $5000. Ann's income is $60,000.

Betty lives in the Appleville suburb, where the distribution is also approximately normal, but with mean $40,000 and standard deviation $10,000. Betty's income is $45,000.

a) For each woman, find the **z-score** of her income (in relation to her suburb).
Who is richer compared to ***her own neighbors***?



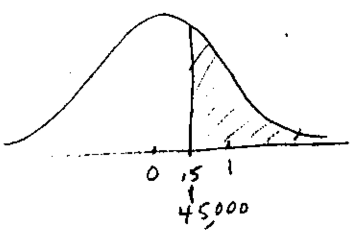


$z = \frac{60000-65000}{5000} = \frac{-5000}{5000} = -1$

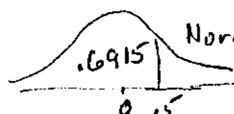
$z = \frac{45{,}000-40{,}000}{10{,}000} = \frac{5000}{10{,}000} = .5$

Betty is .5 s.d.'s above the mean in Appleville, Ann is 1 s.d. below, in Plumville. Betty is richer.

b) What proportion of the incomes in **Appleville** are ***higher*** than Betty's?



Betty's z = .5 (part a)

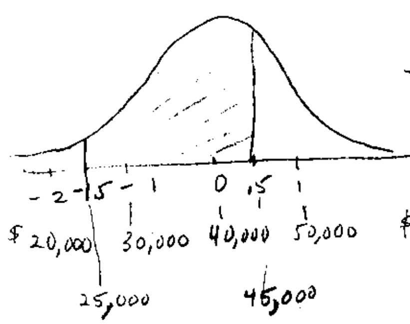


Proportion below {x = 45,000, z = .5} is .6915, Proportion above = 1 - prop. below

1.0000
-.6915
.3085

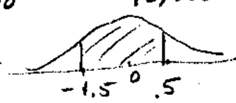
About 31% are higher

c) What proportion of the incomes in **Appleville** are between $25,000 and $45,000?



for 25,000, $z = \frac{25{,}000-40{,}000}{10{,}000} = \frac{-15{,}000}{10{,}000} = -1.5$

Proportion between z's -1.5 & .5



= Prop. below z = .5 = .6915 from (b)

− Prop below z = −1.5 = .0668 from table

Prop. between = .6247 About 62.5%

d) Appleville decides to give 20 pounds of Apples free to the poorest 20% of its occupants. What is the cutoff income level for getting free apples?

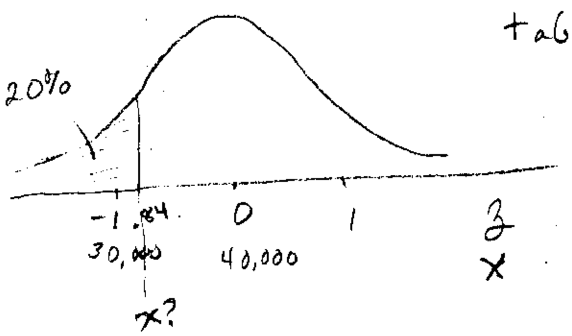


table: z = -.84 has .2005 to the left of it.

Change z-score of -.84 to dollars!

formula $x = \mu + z\sigma = 40{,}000 + (-.84)\cdot 10{,}000$
$= 40{,}000 - 8{,}400$
$= \$31{,}600$

other formula $z = \frac{x-\mu}{\sigma}$ $\quad -.84 = \frac{x - 40{,}000}{10{,}000}$ + solve

Words: The value is .84 s.d.'s below the mean. Since one s.d. is $10,000, it is $8,400 below the mean. The mean is $40,000 so it's at $40,000 - $8,400, or $31,600.

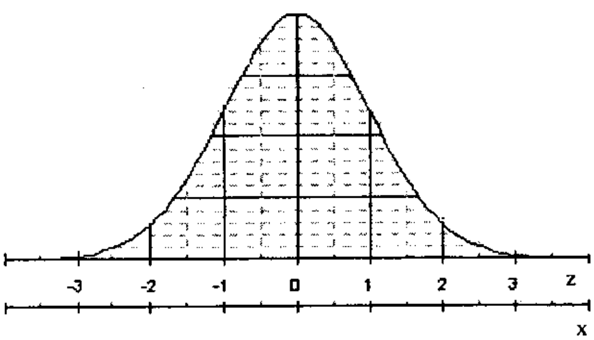
DO IT!

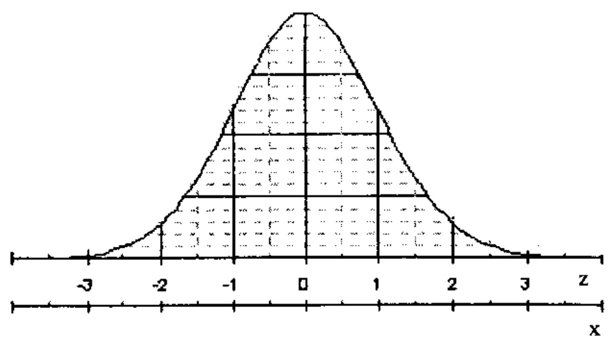
First decide: We need to go from what to what?
Mark each number you use *raw, (z,) [area/proportion] *** what are the raw units?**

In **Samuel Adams** high school, the heights of senior women is known to have a roughly normal distribution for the general population, with a **mean of 66 inches** and a **standard deviation of 3 inches.**.

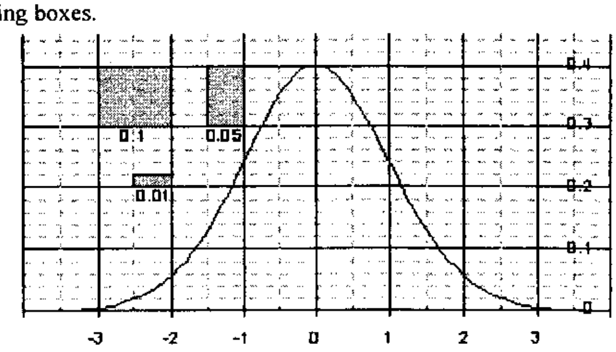
In **Vespucci** high school, the heights of senior women also have a normal distribution, but with a **mean of 63** and a **standard deviation of 2 inches.**

a) Who is "taller" relative to her own school peer group----Mary at Samuel Adams, who is **66.5** inches tall, or Alice at Vespucci who is **65** inches tall?
Find the z-**scores, and use them to answer...**





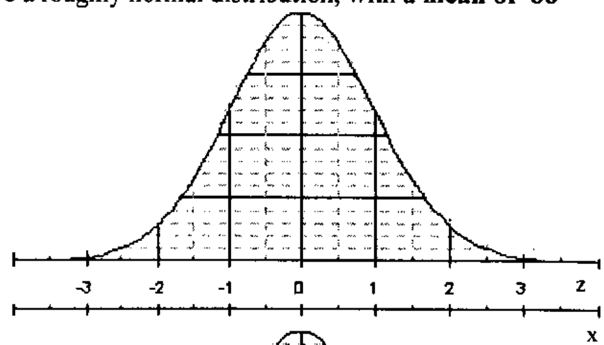
b) What **Percentile** is **Alice (65 inches) at Vespucci** at? Shade the picture. Try it two ways, using the Normal table, and using the 68-95-99.7 rule. You can also check, approximately, by counting boxes.

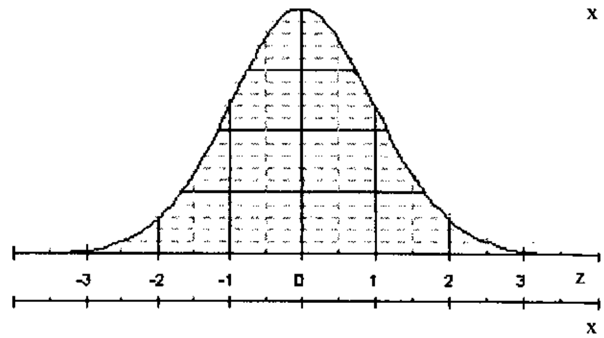


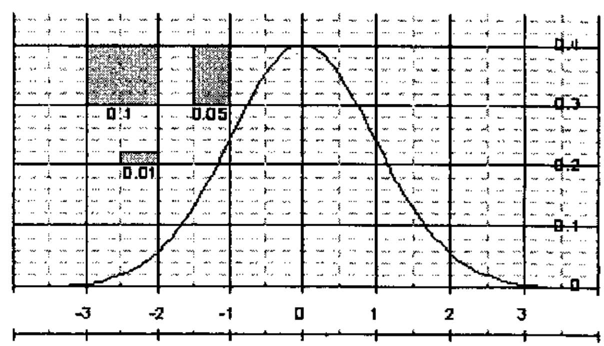
c) At **Vespucci,** for the senior play, the benighted theater teacher refuses to cast as "leading lady" anyone who is taller than the "leading man"-- who is only 65 inches tall. **What percent** of the senior women are NOT eligible? (Make your own sketch, or use another on the page)

In **Samuel Adams** high school, the heights of senior women have a roughly normal distribution, with a **mean of 66 inches** and a **standard deviation of 3 inches**..

d) The choir robes at Sam Adams were bought some years back, to fit women **between the heights of 60" and 67"**. What proportion of the school will fit the choir robes?







e)At Samuel Adams, they traditionally give the tallest 10% of the senior women flags to carry in the graduation procession. What is the height requirement for carrying a flag?